

C.U.SHAH UNIVERSITY

Summer Examination-2017

Subject Name : Numerical Methods

Subject Code : 4SC04MTE1

Branch: B.Sc.(Mathematics,Physics)

Semester : 4

Date : 20/04/2017

Time : 10:30 To 01:30

Marks : 70

Instructions:

- (1) Use of Programmable calculator & any other electronic instrument is prohibited.
- (2) Instructions written on main answer book are strictly to be obeyed.
- (3) Draw neat diagrams and figures (if necessary) at right places.
- (4) Assume suitable data if needed.

- Q-1 Attempt the following questions (14)**
- a) The _____ method has a fast rate of convergence. (01)**
- (a) Bisection method (b) False position method
(c) Newton Raphson method (d) none of these
- b) The modified Euler's method is the Runge-Kutta method of _____ order. (01)**
- (a) 3rd (b) 1st (c) 4th (d) 2nd
- c) Out of four Runge-Kutta methods, the Runge-Kutta method of _____ order is most commonly used in applications. (01)**
- (a) 3rd (b) 1st (c) 4th (d) 2nd
- d) Write Picard's formula for $\frac{dy}{dx} = f(x, y)$ with $f(x_0) = y_0$. (01)**
- e) What is the value of $f'(x)$ in general by Stirling interpolation formula? (01)**
- f) Write Simpson's one third rule. (01)**
- g) Write n^{th} approximation of iteration method. (01)**
- h) Give value of $f'''(x)$ by Newton's forward interpolation formula. (01)**
- i) Give general formula for Modified Euler Method. (02)**
- j) Give value of a & b such that root of $f(x) = 0$ lies between a & b , where $f(x) = x^2 + x - 5$. (02)**
- k) Find the value of $\int_0^1 e^x dx$ with $h = 1/2$ by Trapezoidal rule. (02)**

Attempt any four questions from Q-2 to Q-8.

- Q-2 Attempt all questions (14)**
- a) Find the positive root of $2x = 3 + \cos x$ by bisection method. (05)**
- b) Find a root of $f(x) = 3x - 6 - \log_{10} x$ using Iteration Method up to four decimal places. (05)**
- c) Find a root of the equation $x^3 - 2x - 5 = 0$ correct up to three significant figures by using the Newton-Raphson method. (04)**



- Q-3 Attempt all questions (14)**
- a) Evaluate $\int_2^6 \log x \, dx$ by Weddle's rule and taking $n=5$. (05)
- b) Evaluate $\int_{0.1}^{0.7} e^x + 2x \, dx$ by Trapezoidal rule and taking $n = 6$. (05)
- c) Evaluate $\int_0^{\frac{\pi}{2}} e^{\sin x} \, dx$ by Simpson's 3/8 rule and taking $n = 6$. (04)

- Q-4 Attempt all questions (14)**
- a) Prove that Newton-Raphson Method has second order convergence. (05)
- b) Apply Taylor's series method to obtain approximate value of y at $x = 0.2$ for the differential equation $\frac{dy}{dx} = 2y + 3e^x, y(0) = 0$. (05)
- c) Find a root of the equation $\cos x - xe^x = 0$ correct up to three decimal places by using the False-position method. (04)

- Q-5 Attempt all questions (14)**
- a) Find the solution of $\frac{dy}{dx} = e^x - y$ up to the fifth approximation. Using Picard's method given that $y(0) = 0$. (07)
- b) Apply Milne's method to find the solution of the differential equation $\frac{dy}{dx} = x - y^2$ in the range $0 \leq x \leq 1$. (07)

- Q-6 Attempt all questions (14)**
- a) Find $f'(0)$ from the following data: (07)
- | | | | | | |
|--------|-----|----|-----|-------|-------|
| x | 3 | 5 | 11 | 27 | 34 |
| $f(x)$ | -13 | 23 | 899 | 17315 | 25606 |
- b) Derive $f'(x)$ by Newton's Forward Interpolation Formula. (07)

- Q-7 Attempt all questions (14)**
- a) Determine $y(0.1)$ and $y(0.2)$ correct to four decimal places from $\frac{dy}{dx} = 2x + y, y(0) = 1$. Use fourth order Runge-Kutta method. (07)
- b) Using Euler modified method, obtain a solution of $\frac{dy}{dx} = x + |\sqrt{x}|, y(0) = 1$ for the range $0 \leq x \leq 0.6$ in steps of 0.2. (07)



Q-8

Attempt all questions

(14)

- a) Find the value of $f'(0.5)$ and $f''(0.5)$ using Stirling's formula from the following data

(07)

x	0.35	0.40	0.45	0.50	0.55	0.60	0.65
y	1.521	1.506	1.488	1.467	1.444	1.418	1.389

- b) The table given below reveals the velocity v of a body during the time t specified. Find its acceleration at $t = 1.1$

(07)

t	1.0	1.1	1.2	1.3	1.4
v	43.1	47.7	52.1	56.4	60.8

